



PRELIMINARIES

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SCA CONTROL - Control systems for your processes

1 Introduction

In this report all the parameters used in other documents are defined. We also focus on the reasoning behind the controllers' design.

The controllers are designed with a "direct method", which means that the sampling/holding effects are taken into account from the beginning and all the operations are performed in the discrete-time framework. This approach leads to several advantage:

- **Accurate Representation:** the direct method inherently considers the discrete nature of the system, ensuring that the controller design accurately reflects the system's behavior at each sampling instance.
- **Avoids Discretization Errors:** there are no additional errors introduced by the discretization process since the design is done directly in the discrete-time domain. This can lead to better performance in some cases.
- **Tailored for Digital Implementation:** the controller is directly designed for digital hardware, making the implementation more straightforward and potentially more efficient.
- **Flexibility in Handling Constraints:** it is easier to incorporate digital constraints (e.g., quantization effects, limited word length) during the design process, potentially leading to a more robust controller in practical implementations.

2 PID controller

We recall the formulation of the PID controller in continuous-time:

$$u(t) = \bar{K}_P e(t) + \bar{K}_I \int_0^t e(i) di + \bar{K}_D \frac{d}{dt} e(t) \quad (1)$$

An intuitive method to obtain a discrete version of the PID is the following:

$$u(k) = K_P e(k) + K_I \sum_{i=0}^k e(i) + K_D (e(k) - e(k-1)) \quad (2)$$

where $K_P = \bar{K}_P$, $K_I = \bar{K}_I T_s$, $K_D = \bar{K}_D / T_s$, and T_s is the sampling period. We express the PID controller in $\mathcal{Z}$ -transform:

$$C(z) = K_P + K_I \frac{z}{z-1} + K_D \frac{z-1}{z} = \frac{K_1 z^2 - K_2 z + K_D}{z(z-1)} \quad (3)$$

where $K_1 = K_P + K_I + K_D$ and $K_2 = K_P + 2K_D$. Finally, we define the parameter $\alpha = K_1 / K_2$ that will be used later on.

3 Time specifications

The controllers are designed in such a way the step response of the resulting closed-loop transfer function satisfies the following time specifications:

- **Overshoot:** it is the amount by which the system's response exceeds the desired final value. It is often associated with the "quality" of control because it indicates how well the system achieves the desired reference without excessive deviation. Lower overshoot generally implies a smoother, more precise response, which is often desirable in high-quality control systems.
- **Settling Time:** it is the time it takes for the system's response to remain within a certain percentage (usually 2% or 5%) of the final value and stay there. It essentially measures how quickly the system stabilizes after a disturbance or a change in input. Settling time can be linked to the "aggressiveness" of the control system. A shorter settling time means the system responds quickly and aggressively to changes, reaching stability faster. However, this might come at the cost of increased overshoot or oscillations.

Clearly, in order to compare the control of PID and AC, the controllers must be designed with the same values of overshoot and settling time.

4 Design parameters

When designing the controllers, the following parameters are defined:

- $\alpha = K_1/K_2$ (just for PID): it is the third constraint (apart from overshoot and settling time) to match the three degrees of freedom of PID. If α increases, the integral gain K_I increases with respect to the derivative gain K_D , and vice versa.
- $\beta = \frac{1/T_s}{\text{Bandwidth of the process}}$: it is linked to the choice of the sampling period. Greater values of β indicate smaller sampling periods with respect to the dynamics of the process, and imply smoother responses.
- $\gamma = \frac{\text{Settling time of the process}}{\text{Settling time}}$: it generates the settling time specification based on the process dynamics. Bigger values of γ implies smaller settling times, so more aggressive controllers.
- *overshoot*: defined above. It is meaningful regardless the dynamics of the process to control.

5 Control architectures

By default the theoretical study of AC controller compared with PID is performed using the simple feedback loop with one-degree-of-freedom (1-DoF) structure. The block diagram is shown in Fig. 1. Instead, in the practical examples where we apply both PID and AC controllers, also two-degree-of-freedom (2-DoF) structure is also used to improve the final result (Fig. 2). Specifically, 2-DoF structure is often used to improve the set-point response or to reduce the effect of measurable disturbances.

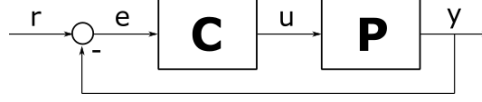


Figure 1: Block diagram for 1-DoF feedback loop control.

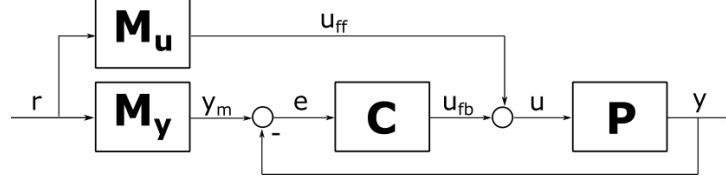


Figure 2: Block diagram of 2-DoF feedback loop control.

6 Process structure

AC controller is analytically designed from the process that is characterized by the following structure parameters:

- number of poles
- number of zeros
- number of delay samples

In the analysis of AC controller we will use different type of process structures. For simplicity, we will use the following notation: if we want to denote processes with two poles and one zero, we will use the abbreviation "2p1z-processes".

Regarding the number of delay samples of the process, we will use the symbol d .

References

- [1] G. F. Franklin, J. D. Powell, M. L. Workman, *Digital Control of Dynamic Systems*, Pearson, 2015.
- [2] K. Ogata, *Discrete-Time Control Systems*, Prentice Hall, 2010.
- [3] N. S. Nise, *Control Systems Engineering*, Wiley, 2020.
- [4] K. J. Astrom, Tore Hagglund, *Advanced PID Control*, ISA-The Instrumentation, Systems, and Automation Society, 2006.

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