OUTPUT NOISE ANALYSIS

ENGLISH

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SCA CONTROL - Control systems for your processes

1 Introduction

Output noise (or measurement noise) can generate undesired control actions, leading to actuator wear and reduced performance. The effects of measurement noise can be alleviated by filtering the measurement signal. However, heavy filtering likely degrades the performance of the controller. Consequently, reducing the need for filtering requires the controller to be robust against measurement noise. The aim of this paper is to show that AC controllers provide greater robustness to measurement noise compared with PID controllers.

2 Theoretical background

Suppose that the reference $r(k)$ is constant and the output $y(k)$ has converged to the regime value $\bar{y}$. Then, suppose that $y(k)$ is affected by white noise $n(k)$ with variance σ_y^2. Therefore, $y(k)$ can be modeled as a WSS (wide-sense stationary) Gaussian Process characterized by the following parameters:

- mean $\mathbb{E}[y(k)] = \bar{y}$
- variance $var[y(k)] = \sigma_y^2$
- statistical power $M_x = \bar{y}^2 + \sigma_y^2$
- signal power $P = \frac{1}{N}\sum_{k=1}^{N} y(k)^2$. It is related to the electrical power, the exact relationship depends on the actuation nature. Using the Law of Large Number, it follows that P converges to M_x as N increases.

To compensate the measurement noise, the controller generates a control action which in turn is affected by noise with variance σ_u^2. To analyze the robustness to measurement noise, the Noise Gain is often defined:

$$k_n = \frac{\sigma_u}{\sigma_y} \tag{1}$$

where σ_y and σ_u are the standard deviations of the output and the control action respectively. A smaller k_n indicates a greater robustness to measurement noise.

3 Experimental setup

Having defined the noise gain as a robustness measure, we now describe the experimental setup used to compare AC and PID controllers under varying specifications. For a given process structure, both AC and PID controllers are designed based on specific specifications (α, β...). See "*preliminaries*" document for the explanation of such parameters. Then, white noise with a certain power σ_y^2 is applied to the output and, for both controllers, the noise gain (say $k_{n,AC}$ and $k_{n,PID}$ respectively) is calculated from a large number of samples (10,000). Finally, the ratio between these two parameters is calculated:

$$k_{n,ratio} = \frac{k_{n,PID}}{k_{n,AC}} \tag{2}$$

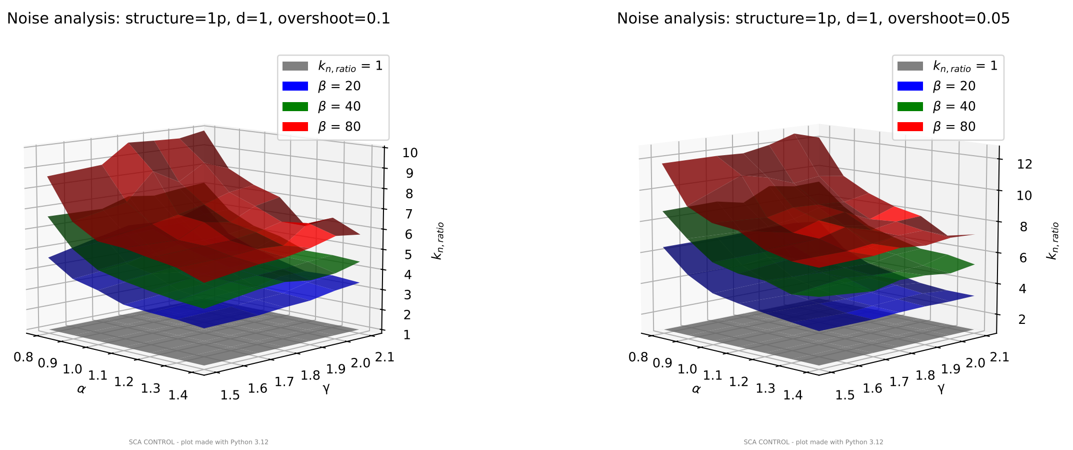
If $k_{n,ratio} > 1$, it means that the AC controller is more robust to measurement noise than PID for this combination of specifications.

The test is then repeated for other values of specifications and all the outcomes are plotted in a graph.

NOTE: it can be shown that $k_{n,ratio}$ does not depend on the noise power or on the process static gain.

3.1 1p-processes

In Fig. 1, we plot the values of $k_{n,ratio}$ for the 1p-process case. As one can observe, $k_{n,ratio}$ increases as α decreases and β increases. It remains almost constant varying γ. Then, $k_{n,ratio}$ slightly increases as the overshoot requirement decreases. Finally $k_{n,ratio}$ is always greater than 1, so the AC controller is always more robust in this case.



(a) (b)

Figure 1: Test for 1p-process with a) 10% overshoot, b) 5% overshoot.

3.2 1p1z-processes

In Fig. 2, we plot the values of $k_{n,ratio}$ for the 1p1z-process case. The results are similar to the 1p-process case. Therefore, $k_{n,ratio}$ increases as α decreases and β increases. It remains almost constant varying γ. Then, $k_{n,ratio}$ slightly increases as the overshoot requirement decreases. Finally $k_{n,ratio}$ is always greater than 1, so the AC controller is always more robust in this case.

3.3 2p-processes

In Fig. 3, we plot the values of $k_{n,ratio}$ for the 2p-process case[1]. Regarding the case of real-coincident poles (Fig. 3.a and Fig. 3.b), $k_{n,ratio}$ increases as γ increases. It remains almost constant varying β. Then, $k_{n,ratio}$ slightly increases as the overshoot requirement decreases. Finally $k_{n,ratio}$ is always much greater than 1, so the AC controller is always

[1]For 2p-processes and 2p1z-processes it was not possible to design the PID controller as a function of α, so the dependency on α is not considered.

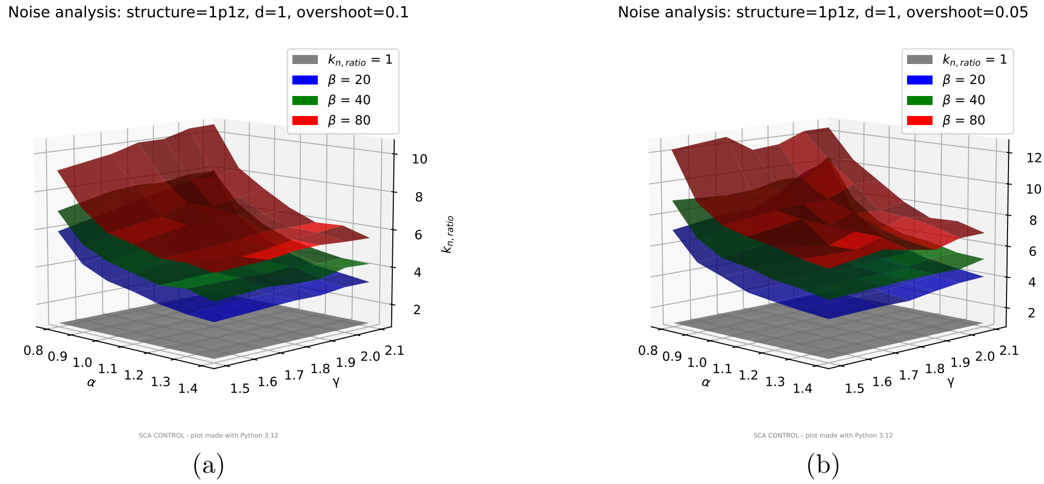


Figure 2: Test for 1p1z-process with a) 10% overshoot, b) 5% overshoot.

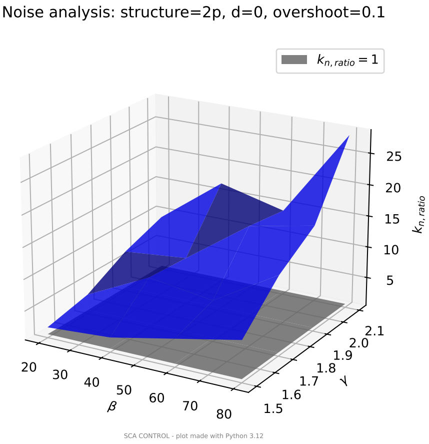
much more robust in this case. Regarding the case of complex-conjugate poles (Fig. 3.c), the considerations are the same, with greater values of $k_{n,ratio}$, therefore with even more robustness for the AC controller as result.

3.4 2p1z-processes

In Fig. 4, we plot the values of $k_{n,ratio}$ for the 2p1z-process case. The results are similar to the 2p-process case. Therefore, $k_{n,ratio}$ increases as γ increases. It remains almost constant varying β. Then, $k_{n,ratio}$ slightly increases as the overshoot requirement decreases. Finally $k_{n,ratio}$ is always much greater than 1, so the AC controller is always much more robust in this case.

4 Conclusion

The experiments suggest that, in general, the AC controller is more robust to output noise. This is particularly evident for 2p-process and 2p1z-process cases. The reason is the impulsive shape of the control action step response. Indeed, for the 2p-process and 2p1z-process experiments, the design of PID controller was possible with small values of α that implies a greater derivative contribution (see "*preliminaries*" document). In contrast, the much smoother step response of the AC control action leads to a greater robustness to output noise. As a result, the use of the AC controller can guarantee lower actuation wear without degrading the performance of the controller.



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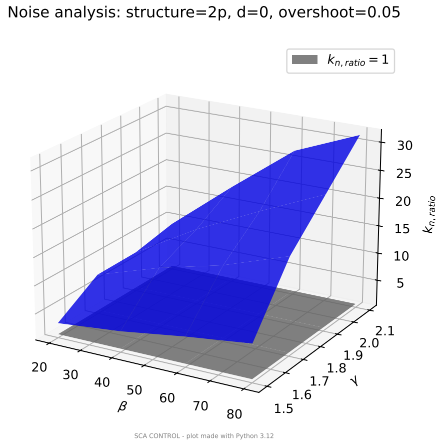

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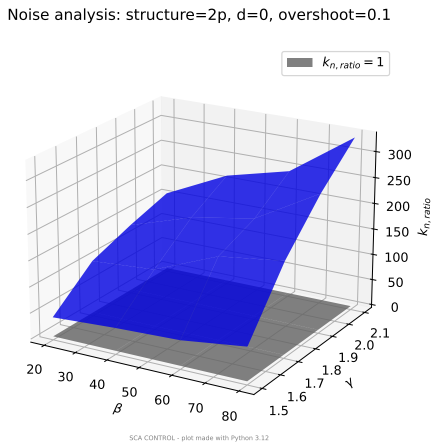
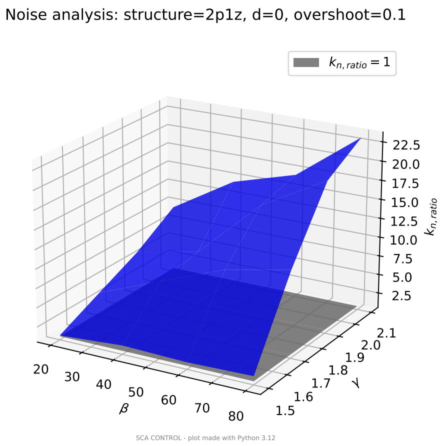
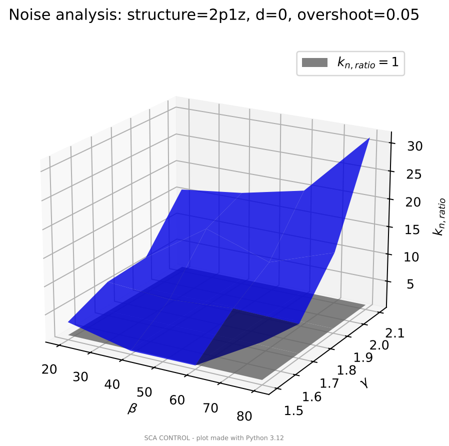


(c)

Figure 3: Test for 2p-process with a) 10% overshoot and real and coincident poles, b) 5% overshoot and real-coincident poles, c) 10% overshoot and complex-conjugate poles.



(a)



(b)

Figure 4: Test for 2p1z-process with a) 10% overshoot, b) 5% overshoot.

References

[1] P. Cuff, *ELE 301: Signals and Systems*, Princeton University, 2011-12.

[2] V. R. Segovia, T. Hagglund, K. J. Astrom, *Measurement noise filtering for PID controllers*, Department of Automatic Control, Lund University, 2014.

[3] V. R. Segovia, T. Hagglund, K. J. Astrom, *Design of Measurement Noise Filters for PID Control* , Department of Automatic Control, Lund University, 2014.

Contacts

For more information, do not hesitate to contact SCA CONTROL through:

- E-mail: support@scacontrol.com
- Phone: +39 3429411838
- Website: www.scacontrol.com