ROBUSTNESS ANALYSIS

ENGLISH

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SCA CONTROL - Control systems for your processes

1 Introduction

In this paper we compare AC and PID controllers in terms of robustness to instability and to process model uncertainties. We expect PID to guarantee better robustness, since AC design is strictly dependent on the process model. If this is verified, the aim is to quantify the lack of robustness of AC with respect to PID, and whether this could compromise control system performance.
For the analysis of robustness to instability, we consider:

- Phase margin (P_m): for the analysis of robustness to instability to phase variations
- Gain margin (G_m): for the analysis of robustness to instability to gain variations
- Peak Sensitivity (M_S): for the analysis of robustness to process model uncertainties

2 Theoretical background

Regarding the analysis of robustness to instability, we consider the open-loop transfer function $L(z) = C(z)G(z)$, where $C(z)$ is the transfer function of the controller and $G(z)$ the transfer function of the process.
The phase margin P_m is defined as:

$$P_m = 180° + \phi(\omega_{cp}) \tag{1}$$

where $\phi(\omega_{cp})$ is the phase lag of $L(z)$ at crossover frequency ω_{cp}, at which the gain is unitary (0 dB). Phase margin is a stability measure in control systems, representing the amount of phase shift needed to reach $-180°$ degrees at the gain crossover frequency. A larger phase margin indicates a more stable system, while a negative phase margin suggests instability and potential oscillations. It is generally required that P_m lies between 30° and 60°, with 45° being a common target.
The gain margin G_m is defined as:

$$G_m[dB] = -M(\omega_{cg}) \tag{2}$$

where $M(\omega_{cg})$ is the gain of $L(z)$ (in dB) at frequency ω_{cg} at which the phase lag is equal to 180°. Gain margin is a stability measure representing how much the system's gain can be increased before it becomes unstable. It is essentially a safety buffer, indicating how much leeway there is before the system starts to oscillate or diverge. A higher gain margin means a more stable system. It is generally required that G_m be between 6 and 12 dB, with 10 dB being typical.
Regarding the analysis to process model uncertainties, we consider the closed-loop transfer function:

$$W(z) = \frac{C(z)G(z)}{1 + C(z)G(z)} \tag{3}$$

We differentiate $W(z)$ with respect to $G(z)$:

$$\frac{dW(z)}{dG(z)} = \frac{d}{dG(z)}\left(\frac{C(z)G(z)}{1 + C(z)G(z)}\right) = \frac{C(z)}{(1 + C(z)G(z))^2} = S\frac{W(z)}{G(z)} \tag{4}$$

where

$$S(z) = \frac{1}{1 + C(z)G(z)} \tag{5}$$

is the sensitivity function. The sensitivity function also describes the transfer function from external disturbance to process output. To quantify the robustness to process model uncertainties we consider the nominal sensitivity peak M_S:

$$M_S = \max_\omega |S(j\omega)| = \max_\omega \left| \frac{1}{1 + C(j\omega)G(j\omega)} \right| \tag{6}$$

A smaller value of M_S indicates a better robustness to process model uncertainties. It is often required that M_S is smaller than 6 dB.

3 Experimental setup

Given a particular process structure, both AC and PID controllers are designed for a specific combination of specifications (α, β...). See "*preliminaries*" document for the explanation of such parameters. For both controllers, P_m, G_m, and M_S are computed and compared. The test is then repeated for other values of specifications and all the outcomes are plotted in graphs.
In addition, for each of the three variables, a reference value is highlighted in the graph. It indicates a threshold, beyond which the control system robustness is considered poor and likely unacceptable. In this paper we consider the following values:

- $P_{m,min} = 45°$ (minimum phase margin)
- $G_{m,min} = 10$ dB (minimum gain margin)
- $M_{S,max} = 6$ dB (maximum sensitivity peak)

3.1 1p-processes

In Fig. 1-2-3, we plot respectively the values of P_m, G_m and M_S for the 1p-process case. We can observe that:

- PID's P_m is always greater, but AC's P_m is safely above the threshold.
- With an overshoot requirement of 10%, PID's G_m and AC's G_m have similar values (at equal β). With an overshoot requirement of 5%, AC's G_m is slightly greater.
- M_S's values are slightly greater with AC, but they remain safely below the threshold.

3.2 1p1z-processes

In Fig. 4-5-6, we plot the values of P_m, G_m and M_S for the 1p1z-process case. We can observe that:

- PID's P_m is always greater, but AC's P_m is safely above the threshold.
- AC's G_m is always greater than PID's G_m.

- M_S's values are slightly greater with AC, but they are strongly below the threshold. It is worth mentioning the poor robustness of PID with $\beta = 20$ (i.e. with a smaller sampling frequency), specifically with small α and big γ.

3.3 2p-processes

In Fig. 7-8-9, we plot the values of P_m, G_m and M_S for the 2p-process case. We can observe that:

- PID's P_m is always greater, but AC's P_m is safely above the threshold.
- AC's G_m and PID's G_m have similar values.
- M_S's values are slightly greater with AC, but they remain safely below the threshold.

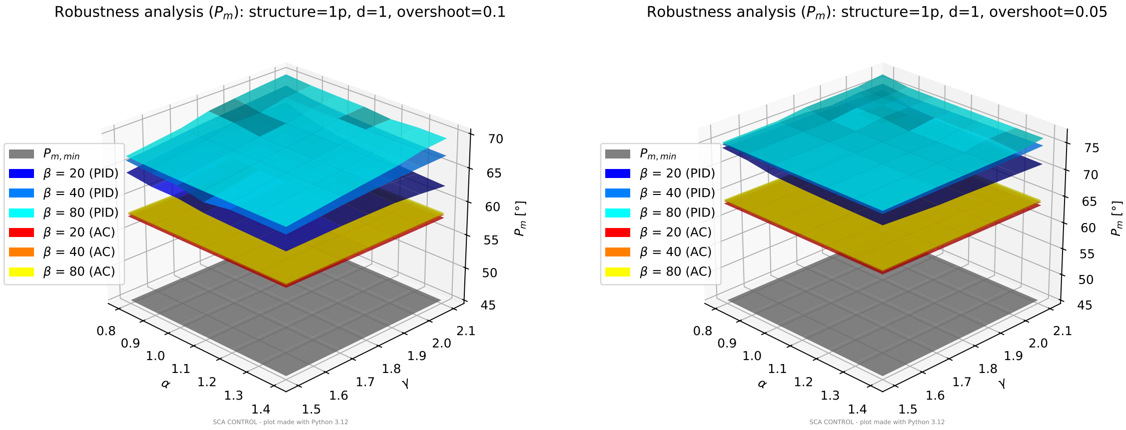
3.4 2p1z-processes

In Fig. 10-11-12, we plot the values of P_m, G_m and M_S for the 2p1z-process case. We can observe that:

- PID's P_m is always greater, but AC's P_m is safely above the threshold.
- AC's G_m is always greater than PID's G_m.
- M_S's values are slightly greater with AC, but they remain safely below the threshold.

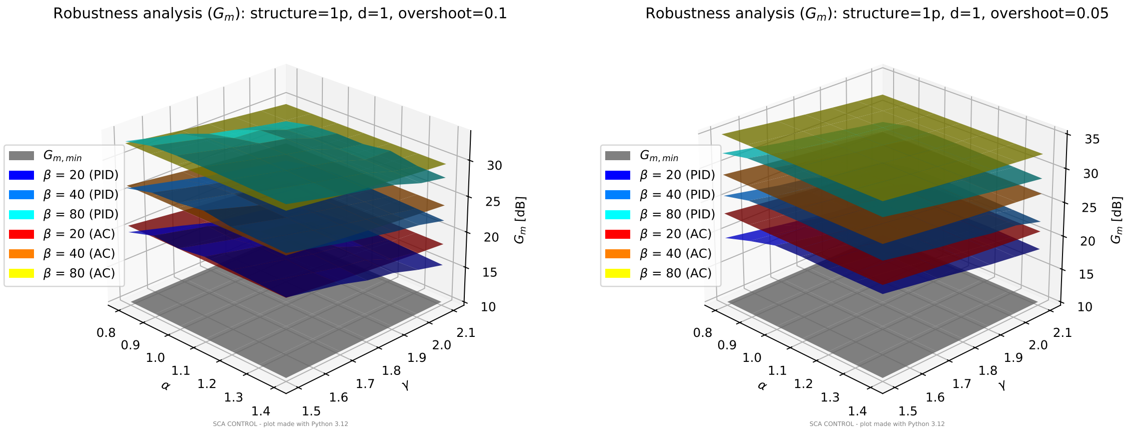
4 Conclusion

The experiments reveal that AC controllers exhibit greater robustness than initially anticipated, particularly in terms of gain margin. Regarding the robustness to instability, AC's P_m is always smaller than PID's, but it exhibits a greater G_m (especially in the cases of processes with a zero).
The main concern was AC's robustness to process model uncertainties. In fact, AC's M_S is always greater than PID's, but always safely below the threshold. In addition, the risk of AC's performance degradation due to process model uncertainties can be mitigated through accurate process parameter identification or by implementing adaptive control when parameters are subject to change.



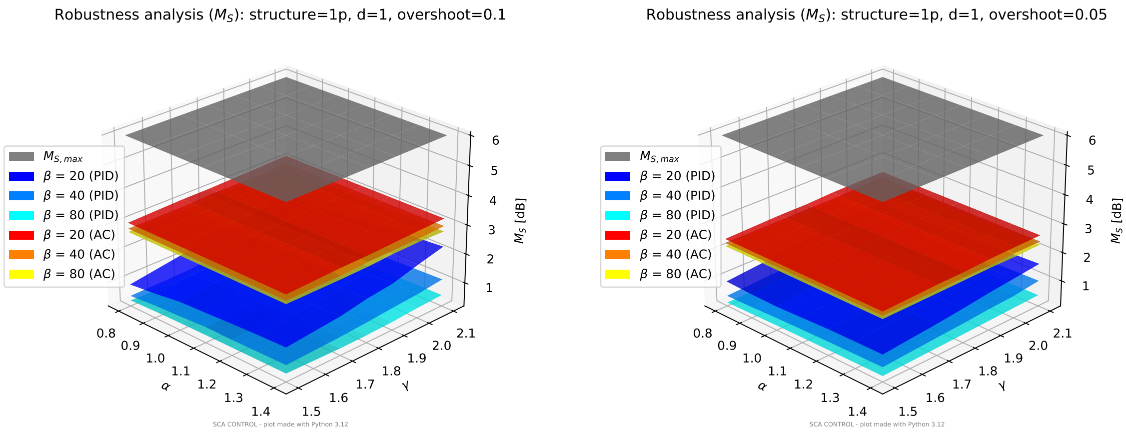
(a) (b)

Figure 1: Test of phase margin for 1p-process with a) 10% overshoot, b) 5% overshoot.



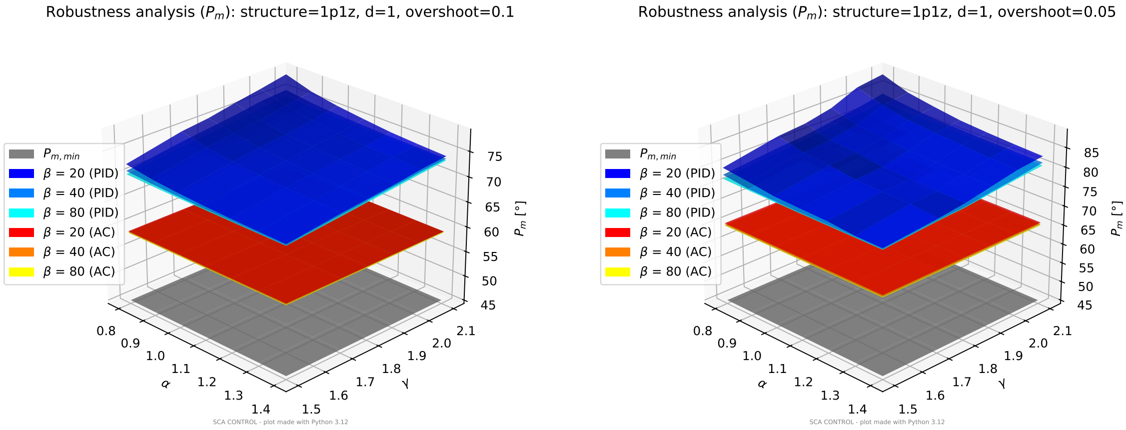
(a) (b)

Figure 2: Test of gain margin for 1p-process with a) 10% overshoot, b) 5% overshoot.



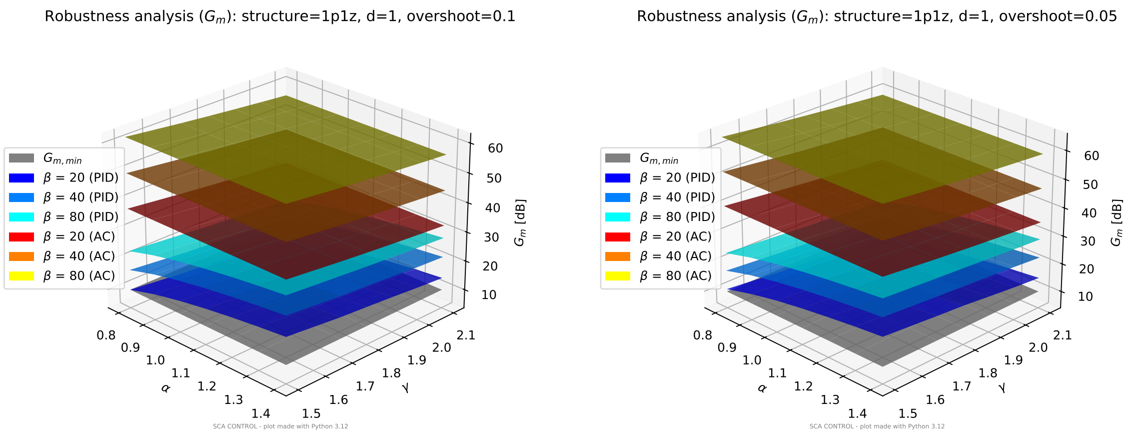
(a) (b)

Figure 3: Test of sensitivity peak for 1p-process with a) 10% overshoot, b) 5% overshoot.



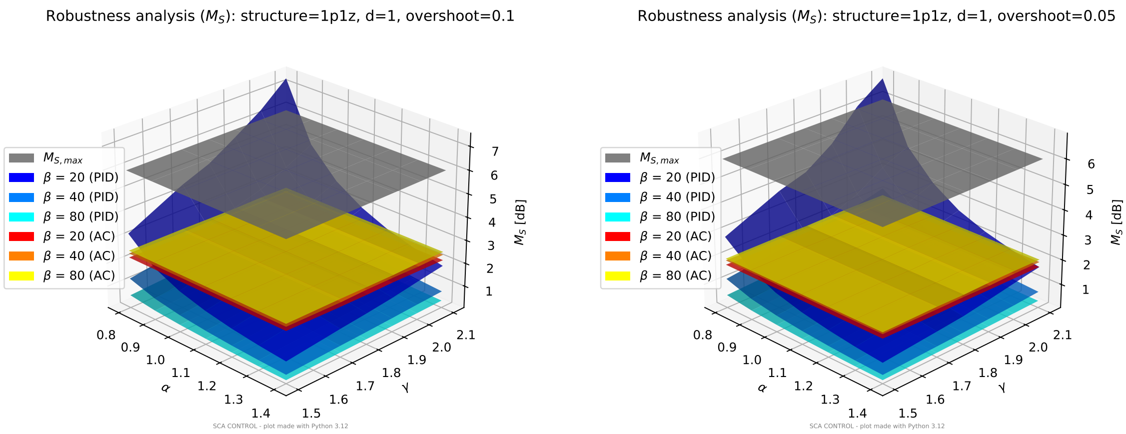
(a) (b)

Figure 4: Test of phase margin for 1p1z-process with a) 10% overshoot, b) 5% overshoot.



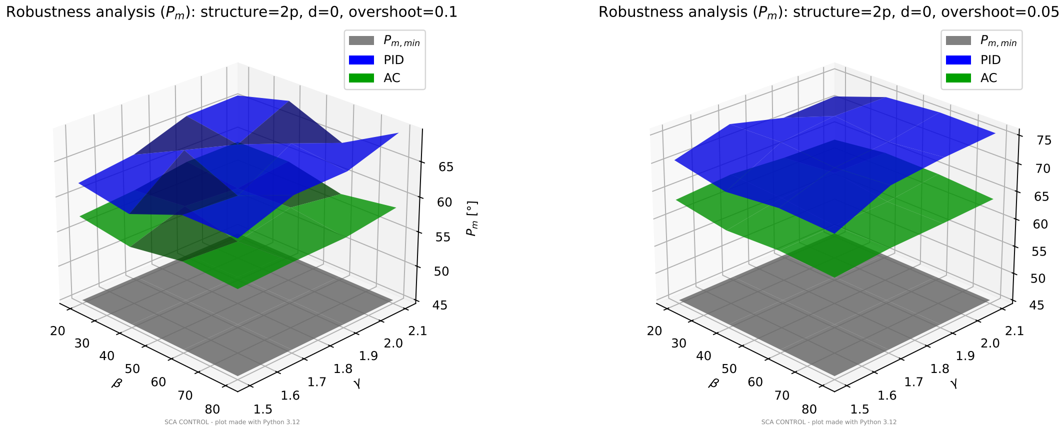
(a) (b)

Figure 5: Test of gain margin for 1p1z-process with a) 10% overshoot, b) 5% overshoot.



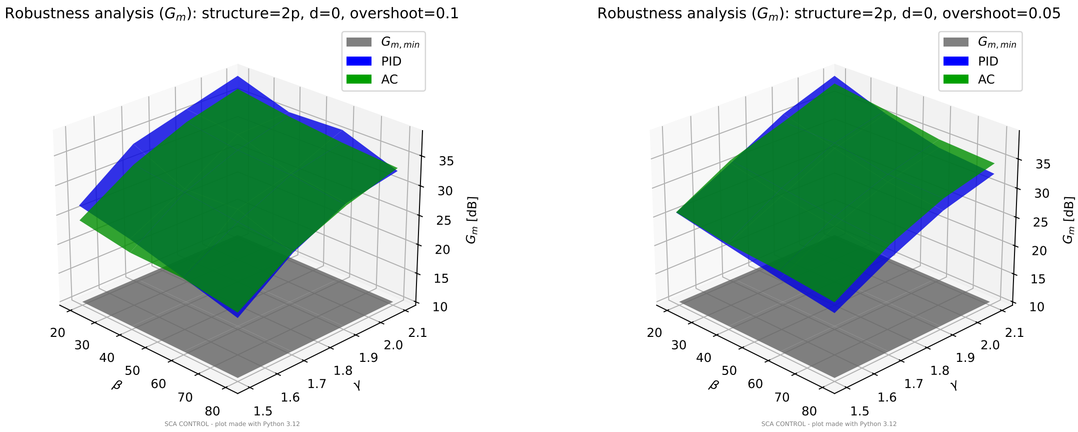
(a) (b)

Figure 6: Test of sensitivity peak for 1p1z-process with a) 10% overshoot, b) 5% overshoot.



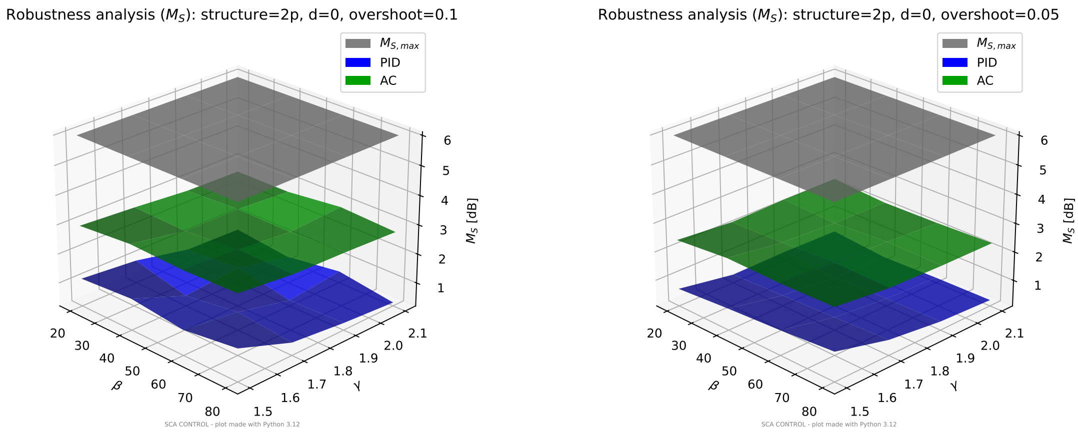
(a) (b)

Figure 7: Test of phase margin for 2p-process with a) 10% overshoot, b) 5% overshoot.



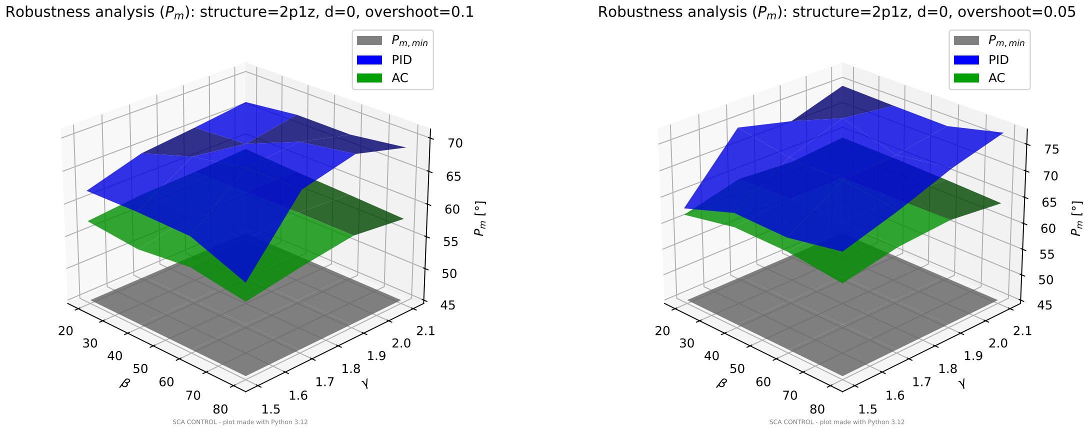
(a) (b)

Figure 8: Test of gain margin for 2p-process with a) 10% overshoot, b) 5% overshoot.



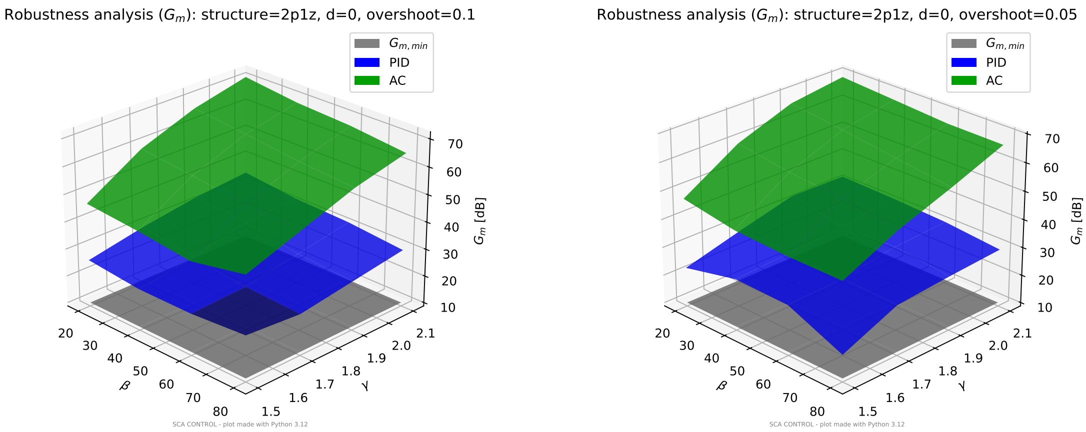
(a) (b)

Figure 9: Test of sensitivity peak for 2p-process with a) 10% overshoot, b) 5% overshoot.



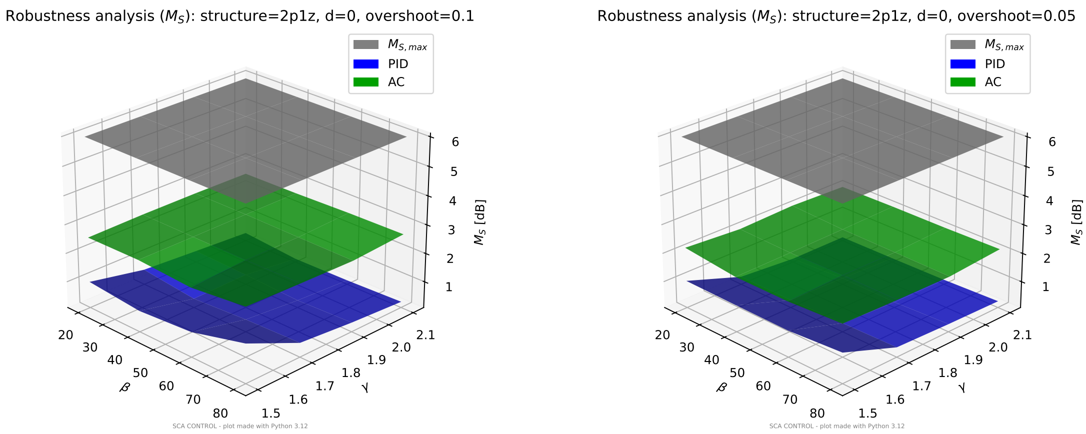
(a) (b)

Figure 10: Test of phase margin for 2p-process with a) 10% overshoot, b) 5% overshoot.



(a) (b)

Figure 11: Test of gain margin for 2p-process with a) 10% overshoot, b) 5% overshoot.



(a) (b)

Figure 12: Test of sensitivity peak for 2p1z-process with a) 10% overshoot, b) 5% overshoot.

References

[1] *CDS 101/110: Lecture 9.1 Frequency Domain Loop Shaping*, California Institute of Technology, 2016.

[2] *Gain and phase margins*, Massachusetts Institute of Technology, 2007.

[3] K. J. Astrom, R. M. Murray, *Feedback Systems, An Introduction for Scientists and Engineers*, Princeton University, 2012.

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